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Dynamics of Collisionless Systems

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by

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Three topics were investigated under this grant. The first area of research was the three-dimensional dynamics of rotating stellar systems. The work entailed analyzing the results of some numerical calculations performed by Dr. F. Hohl of the NASA Langley Research Center. This research was published by Dr. Hohl and myself in "Collapse and Relaxation of Rotating Stellar Systems," *Astron. Jour.*, 84, 585-600 (re-print attached) and in more detailed form by Dr. Hohl, myself and J. Miller in "Collisionless Galaxy Simulations," NASA Reference Publication 1037. This work disclosed that not even the bar-unstable collisionless stellar systems could match the observed velocity profiles of elliptical galaxies. Recently, however, Dr. J. Kormendy of the University of British Columbia compared our numerical results (including some unpublished data) with his own observations of barred spiral galaxies. He concluded that collisionless stellar systems do provide viable models of the bulges in barred spirals.

The second research topic — a joint project with Prof. J. Bardeen of the University of Washington — was a comparison of the various mathematical models of flat galaxies. The crucial choices are gas dynamics vs. stellar dynamics and linear analysis vs. non-linear analysis. Real galaxies, of course, are non-linear, stellar dynamical systems. On the other hand, linear, gas dynamical calculations are the most economical. Stellar dynamical asymptotic theory suggests a crucial difference between physical models which possess an inner Lindblad resonance (ILR) and those which do not. In the latter case waves can propagate through the center of the system, whereas when an ILR is present, inward propagating waves are absorbed at the resonance. This distinction is important because waves which propagate through the center

can feed a powerful instability of flat galaxies. Unfortunately, gas dynamical models do not represent this property correctly — they always permit transmission of waves through the center. Clearly, gas dynamical models, whether linear or non-linear, are likely to be unreliable when they are used to represent a physical system which possesses an ILR. Although gas dynamical models appear plausible when the system has no ILR, the proper analogy between the velocity dispersion in the physical system and the sound speed in the gas dynamical model remains to be determined. Prof. Bardeen and I drew an analogy between the two models based on matching their wave propagation properties in a precise fashion. He performed some linear, gas dynamical calculations and I some non-linear stellar dynamical calculations on analogous models which had no ILRs. The two approaches gave reassuringly similar results. These results will eventually be published, hopefully in collaboration with another individual who has recently developed a reliable method for calculating the non-linear, gas dynamical behavior of flat galaxies.

The last investigation focussed on the effects of self-gravity upon a flat galaxy undergoing a tidal encounter with another galaxy. Toomre and Toomre [Astrophys. Jour., 178, 623-666 (1972)] used a kinematical model (in which self-gravity is neglected) to establish that tidal encounters can produce "bridges and tails" such as those exhibited by the outer region of M51. Their model, however, produced none of the spectacular spiral structure which dominates the inner region of M51.

Conducting a truly dynamical simulation of a tidal encounter involves more than simply adding self-gravity. The real nuisance is finding a stable

initial condition, since flat galaxies are notoriously difficult to stabilize. A stable model is necessary for a clean demonstration of tidal effects because one would like the assurance that any structure which developed during the simulation was truly due to the tidal effects and not to an intrinsic instability of the model. Linear theory calculations by Prof. A. Toomre of M.I.T. and myself [Zang, Ph.D. thesis, M.I.T. (1976)] indicated that a viable initial condition could be based on a galaxy model with a flat rotation curve. A very appealing model is one with half the mass in the disk and the other half in the halo. It has dispersive speeds 50% larger than are required to suppress all axisymmetric instabilities. This model is linearly stable to all angular harmonics and yet linear calculations indicate that it develops pronounced, but transient spiral structure when subjected to tidal forces. (This model is evidently stable in the long run because it possesses an inner Lindblad resonance which eventually damps out the tidally-induced spiral.)

However, when used as the initial condition for an N-body experiment in which no tidal encounter occurred, this model evolved away from the presumably stable initial state. One such calculation is shown in Figure 1. Numerous related simulations were conducted in an effort to account for this behavior. The source of the instability appears to be the rather abrupt outer truncation of the model used in the N-body experiment shown in Figure 1. The mathematical model for the flat rotation curve galaxy extends to infinitely large radii. This poses no problem for the linear analysis, but an N-body simulation requires that the model be truncated at a finite radius. The model shown in Figure 1 was apparently truncated so abruptly that an artificial instability was introduced. (Incidentally, recent linear theory calculations indicate that abrupt truncation does indeed introduce an artificial instability.)



The results of a simulation in which a gentler truncation was employed are shown in Figure 2. The model is still unstable, but much less so.

This slightly unsatisfactory model (no better one is yet known) was then employed as the initial condition for simulations involving a tidal encounter. The cumulative effects of the rapid passage of a companion galaxy were introduced all at once at time  $t = 0$  by merely altering the initial velocities (but not the positions) of all the stars by an amount in accordance with this impulsive tidal approximation. Figure 3 shows the results of a simulation in which the tidal forces reached a maximum of 10% of the equilibrium force. A pronounced and tightly-wound spiral results. This dynamical calculation (which includes self-gravity) contrasts sharply with the kinematical calculation referred to above. An improved calculation is shown in Figure 4. Here the tidal forces are introduced in a more realistic (less symmetric) fashion. Nonetheless, an impressive two-armed spiral results.

These and numerous other simulations will be reported in a future paper.

## LEGENDS

Figure 1: Normal evolution of a flat rotation curve model with an abrupt outer cut-off. A star at the outer edge of the disk takes 16 time units (lower right of each frame) to make one rotation in the counter-clockwise direction.

Figure 2: Normal evolution of a flat rotation curve model with a gentler outer cut-off. This model is less unstable than the preceding one.

Figure 3: Tidal evolution of a flat rotation curve model. The symmetric tidal forces are proportional to position. The outermost stars receive a 10% velocity impulse at  $t = 0$ .

Figure 4: Tidal evolution of a flat rotation curve model. Tidal forces are applied asymmetrically. The outermost stars receive a 20% velocity impulse at  $t = 0$ .

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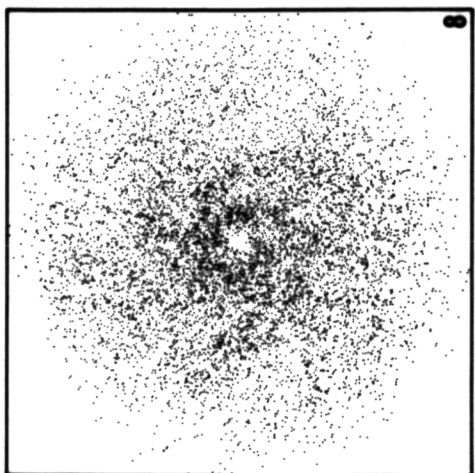
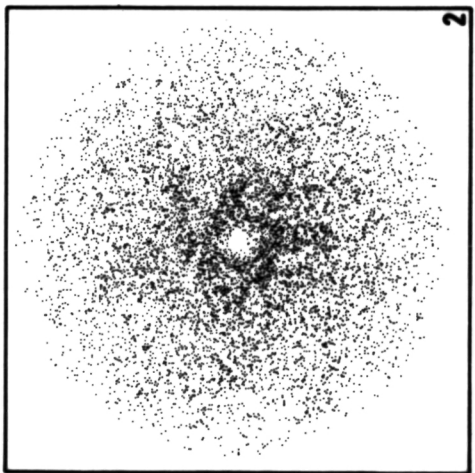
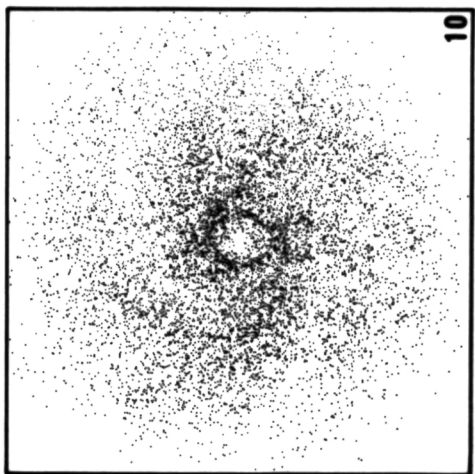
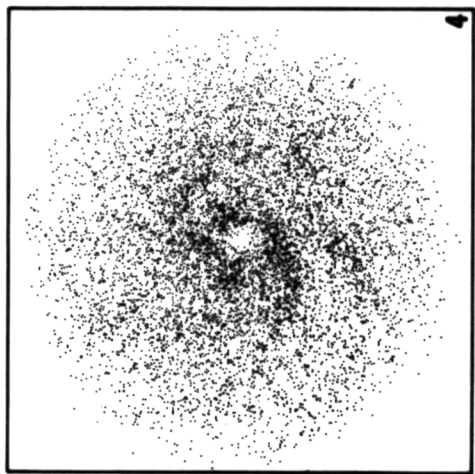
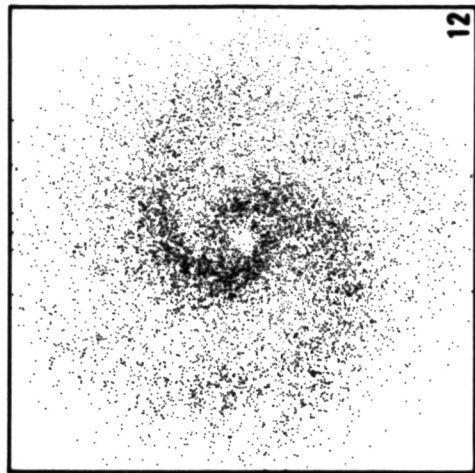
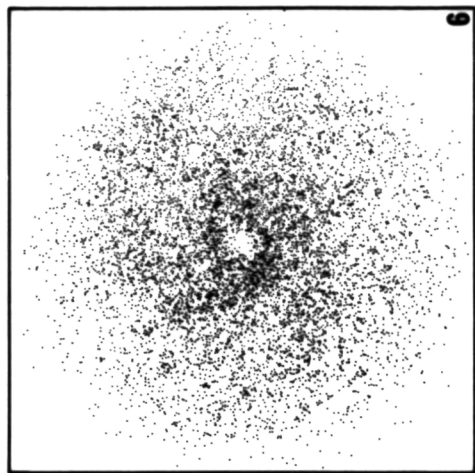


Figure 1



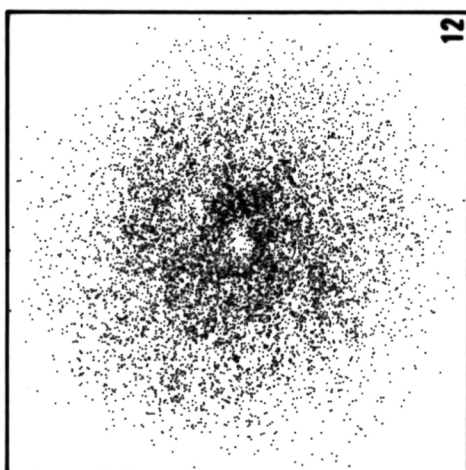
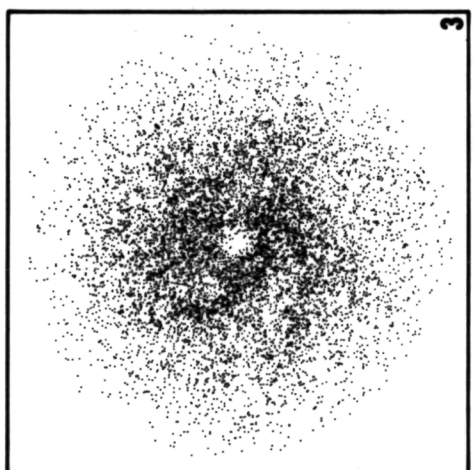
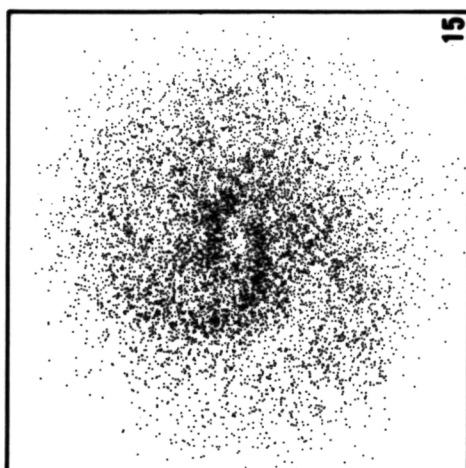
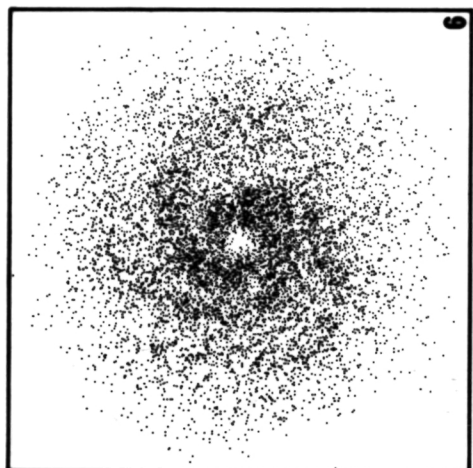
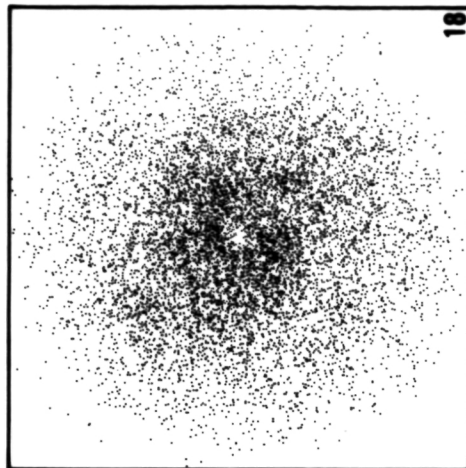
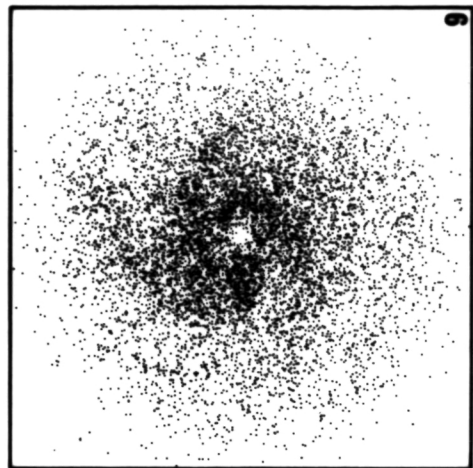


Figure 2

$N=4$   $Q=1.5$   $X=2$  gentle outer taper

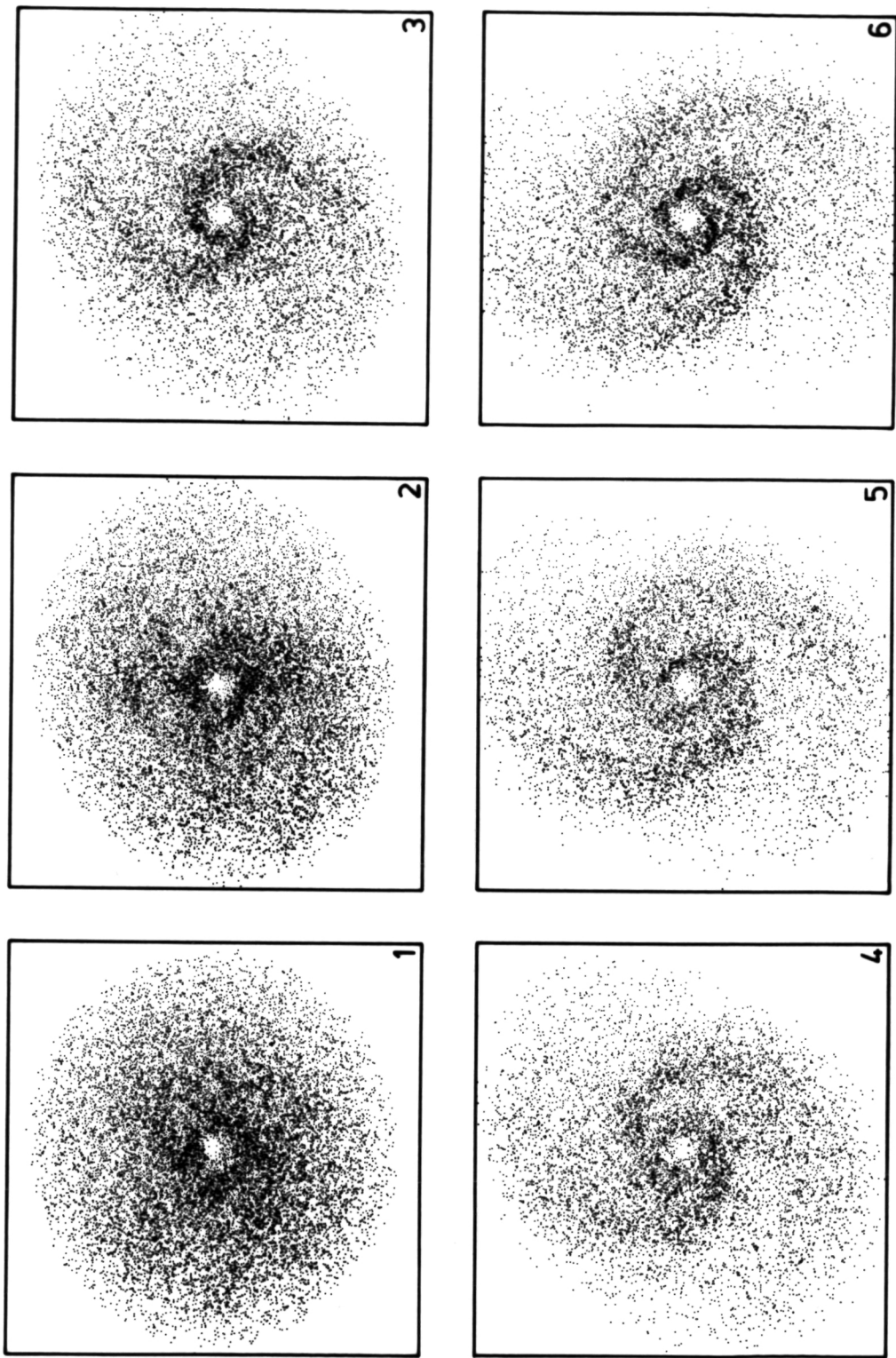


Figure 3



5% tide  $Q=1.5$   $X=2$

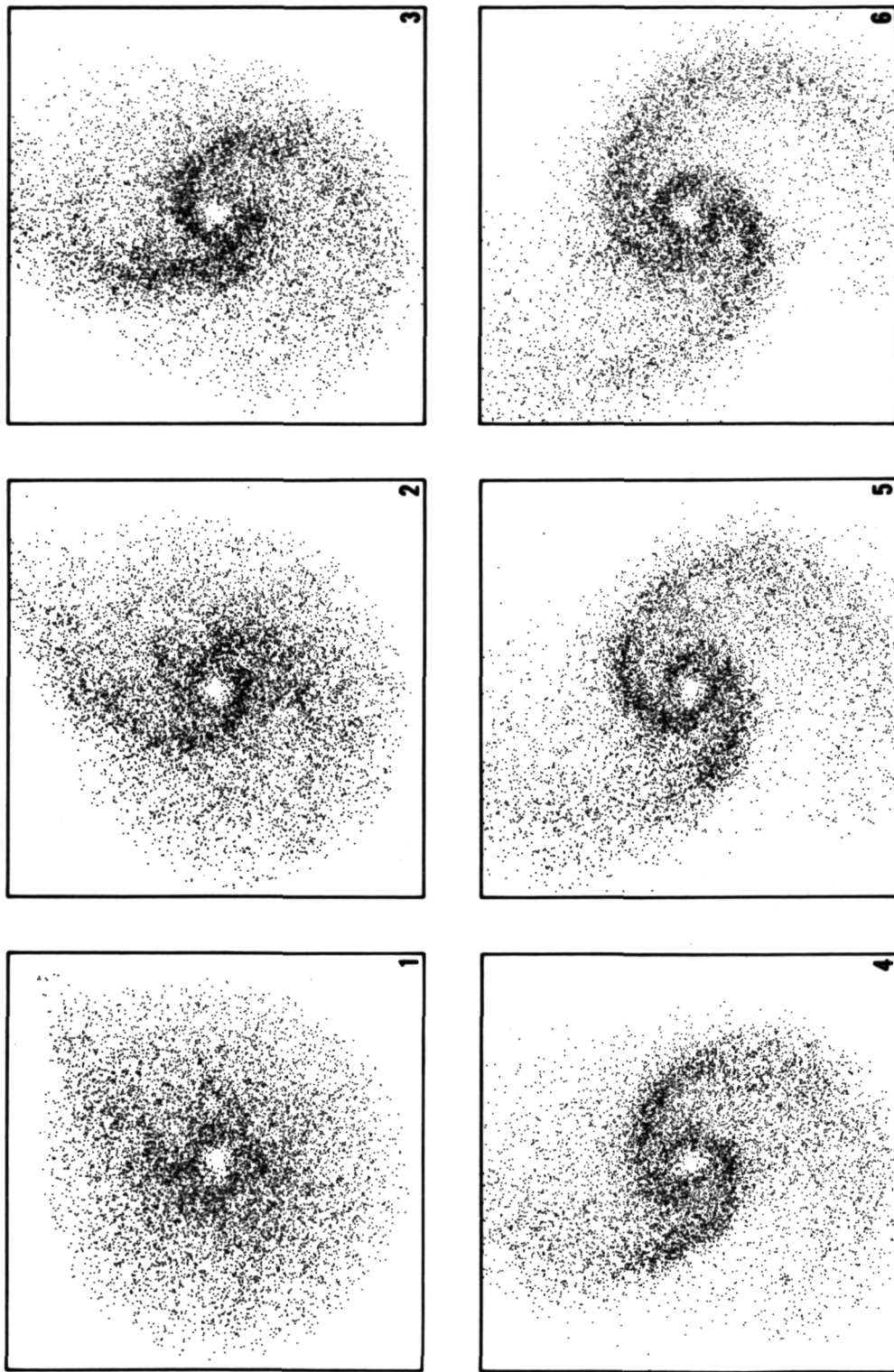


Figure 4

1020 asym. tide  $Q=1.5$   $x=2$   $r_t=8$